

Ch6 p1

MA121

June 14/18

Antiderivatives and the Definite Integral.

6.1 The General Antiderivative

Antiderivatives are the opposite of derivatives.

Given a derivative, what was the original function that generated this derivative?

Simple Example:

If $y = x^2$, $\Rightarrow \frac{dy}{dx} = 2x$ (suppose this is not given)

Suppose we are given $\frac{dy}{dx} = 2x$. First without being given y , we want to ask, "what was the original y that generated $\frac{dy}{dx} = 2x$?"

Some Notation

The anti derivative is sometimes called the integration. To antidifferentiate is also said "to integrate".

Corollary

↳ the antiderivative of a marginal revenue is the revenue function.

↳ the antiderivative of a marginal cost is the cost function.

So, the antiderivative of $y = x^2 + 2$ is $\frac{dy}{dx} = 2x$

The derivative of $y = x^2 + 3$ is $\frac{dy}{dx} = 2x$ as well.

We can see that the antiderivative is not unique.

The one component that makes an antiderivative

not unique is the constant. So to generalize the

antiderivative, we can write that given $\frac{dy}{dx} = 2x$,

its antiderivative form is $y = x^2 + C$, where C

is an arbitrary constant.

Unle.1 p2

MAR21

June 14/8

Important!

Every time we find an antiderivative, we need to write the constant, C . In some cases, the constant may be determined as $\ln C$, or e^C , or any other constant expression.

Notation for Antiderivatives

If $y = f(x)$ and $\frac{dy}{dx} = f'(x)$, we use the $\int$ symbol indicating that we want to antiderive the given expression, i.e. $\int f'(x) dx = f(x) + C$ $\leftarrow$ don't forget the constant
 integrand $\uparrow$ with respect to antiderivative

Basic Rules of Integration, p. 178/179.

$\hookrightarrow \int k dx = kx + C$ where k and C are constants

* $\hookrightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + C$ provided $n \neq -1$.

$\hookrightarrow \int \frac{1}{x} dx = \ln|x| + C$

$\hookrightarrow \int a^{mx+b} dx = \frac{a^{mx+b}}{m \ln a} + C$ $\leftarrow$ only applies if the power is in a linear expression.

Special Case

$\hookrightarrow \int e^{mx+b} dx = \frac{e^{mx+b}}{m \ln e} + C = \frac{e^{mx+b}}{m} + C$

Special case

$\hookrightarrow \int e^{kx} dx = \frac{e^{kx}}{k} + C$

$\hookrightarrow \int k f(x) dx = k \int f(x) dx$

$\hookrightarrow \int f(x) dx \pm \int g(x) dx = \int [f(x) \pm g(x)] dx$

ex.

1 p 181

$\int -6 dx = -6x + C$

to check: $\frac{d}{dx}(-6x + C) = -6$.

4 p 181

$$\begin{aligned} & \int (2x^{-5} + 4x^{-3} - \frac{1}{4}x + 12x^5) dx \\ &= 2 \left[\frac{x^{-5+1}}{-5+1} \right] + 4 \left[\frac{x^{-3+1}}{-3+1} \right] - \frac{1}{4} \left[\frac{x^{2+1}}{2+1} \right] + 12 \left[\frac{x^{5+1}}{5+1} \right] + C \\ &= 2 \left[\frac{x^{-4}}{-4} \right] + 4 \left[\frac{x^{-2}}{-2} \right] - \frac{1}{4} \left[\frac{x^3}{3} \right] + 12 \left[\frac{x^6}{6} \right] + C \\ &= -\frac{1}{2}x^{-4} - 2x^{-2} - \frac{1}{12}x^3 + 2x^6 + C \end{aligned}$$

Ch6.1p3 MA121

June 14/18

Sp181

$$\begin{aligned} & \int (e^{4x} - 2e^{-3x}) dx \\ &= \frac{e^{4x}}{4} - 2 \left[\frac{e^{-3x}}{-3} \right] + C \\ &= \frac{e^{4x}}{4} + \frac{2}{3} e^{-3x} + C \end{aligned}$$

6p181

$$\begin{aligned} & \int (x^2 - 6x)^2 dx \\ &= \int (x^4 - 12x^3 + 36x^2) dx \\ &= \frac{x^{4+1}}{4+1} - 12 \left[\frac{x^{3+1}}{3+1} \right] + 36 \left[\frac{x^{2+1}}{2+1} \right] + C \\ &= \frac{x^5}{5} - 12 \left[\frac{x^4}{4} \right] + 36 \left[\frac{x^3}{3} \right] + C \\ &= \frac{x^5}{5} - 3x^4 + 12x^3 + C \end{aligned}$$

19p181

$$\text{Given } S'(x) = \frac{2400}{3\sqrt{x^2}}$$

$$\Rightarrow S(x) = \int S'(x) dx$$

$$\begin{aligned} \Rightarrow S(x) &= \int \frac{2400}{3\sqrt{x^2}} dx = \int \frac{2400}{x^{2/3}} dx = \int 2400 x^{-2/3} dx \\ &= 2400 \left[\frac{x^{-2/3+1}}{-2/3+1} \right] + C = 2400 \left[\frac{x^{1/3}}{1/3} \right] + C = 7200 x^{1/3} + C \end{aligned}$$

$$\Rightarrow S(x) = 7200 \sqrt[3]{x} + C$$

We are told that when $x=0$, then $S=5000$

$$\text{So, } S(0) = 7200 \sqrt[3]{0} + C = 5000$$

$$\Rightarrow 0 + C = 5000 \Rightarrow C = 5000.$$

$$\therefore S(x) = 7200 \sqrt[3]{x} + 5000$$

When we are given more info, we may be able to find the value of C .

If $x=4$, then

$$S(4) = 7200 \sqrt[3]{4} + 5000$$

$$\approx 16\,429 \text{ cases}$$

Chk. 1p4

MA121

June 14/8

18p 181

$$MR(q) = -0.10q + 80$$

$$\begin{aligned} \text{i) When } q = 100, \quad MR(100) &= -0.10(100) + 80 \\ &= -10 + 80 = \$70. \end{aligned}$$

$$\text{ii) } R(q) = \int MR(q) dq$$

$$\Rightarrow R(q) = \int (-0.10q + 80) dq$$

$$= -0.10 \left[\frac{q^{1+1}}{1+1} \right] + 80q + C$$

$$= -0.10 \left[\frac{q^2}{2} \right] + 80q + C$$

$$\Rightarrow R(q) = -0.05q^2 + 80q + C$$

$$\text{When } q = 0, \text{ then } R = \$0.$$

$$\text{Therefore, } R(0) = 0 \quad \text{or} \quad \Rightarrow -0.05(0^2) + 80(0) + C = 0$$

$$\Rightarrow C = 0. \therefore R(q) = -0.05q^2 + 80q$$

$$\text{When } q = 100,$$

$$R(100) = -0.05(100^2) + 80(100)$$

$$= -0.05(10000) + 8000$$

$$= -500 + 8000 = 7500$$

$$\text{iii) We know that } R = pq, \text{ from } R = -0.05q^2 + 80q$$

$$\Rightarrow R = (-0.05q + 80)(q)$$

$$\text{Thus, the demand function is } p = -0.05q + 80$$

$$\text{When } q = 100, \text{ the price } p \text{ is}$$

$$p = -0.05(100) + 80$$

$$= -5 + 80$$

$$= \$75. \quad \text{or}$$

$$\text{price per unit} = \frac{\text{revenue}}{\# \text{ of units}} = \frac{\$7500}{100} = \$75/\text{unit}.$$